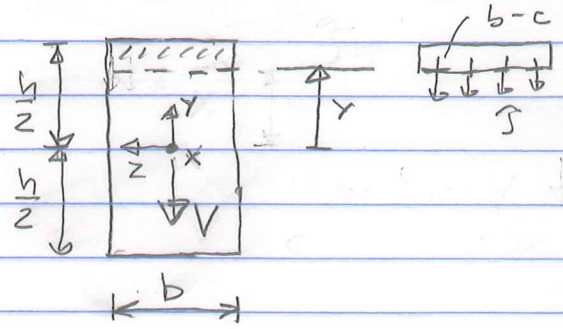
23. Shear stress on symmetric cross-sections
(Linearly elastic, homogeneous, isotropic material)

Example: rectangular section



$$I = \frac{1}{12} h^3 b$$

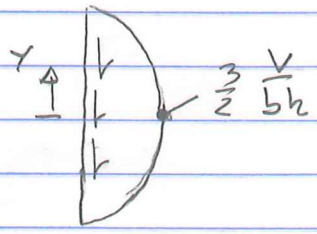
$$Q = b \cdot \left(\frac{h}{2} - y\right) \cdot \left(y + \frac{1}{2}\left(\frac{h}{2} - y\right)\right)$$

$$= \frac{b}{8}\left(h^2 - 4y^2\right)$$

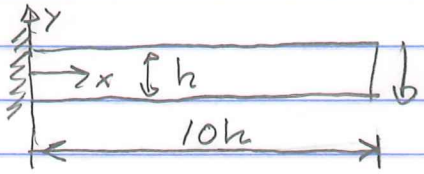
$$\tau_{av} = \frac{VQ}{Ib} = \frac{3}{2}\frac{V}{bh}\left(1 - \frac{4y^2}{h^2}\right)$$

Note for this cross-section, the shear stress along any line parallel to z will be constant at the value of τ_{av}; thus,

$$\tau_{xy} = -\tau_{av}.$$



Compare shear and bending stresses for a cantilever beam.



$V = F$ (constant along length)

$$\tau_{xy} = -\frac{3}{2}\frac{F}{bh}\left(1 - \frac{4y^2}{h^2}\right)$$

$$\tau_{xy,max} = \frac{3}{2}\frac{F}{bh}$$

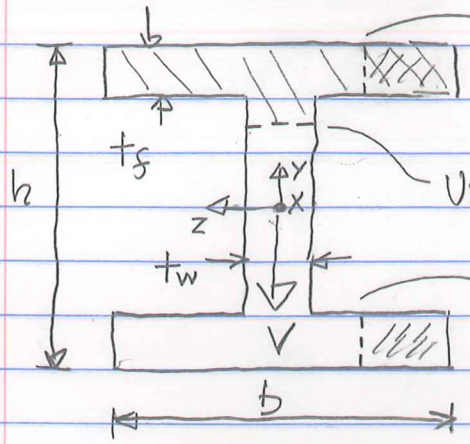
$M_{max} = F \cdot 10h$ at left end

$\sigma_{x,max} = M_{max} \cdot \frac{h}{2} \Big/ \frac{1}{12}h^3 b = 60F/bh$ at left end

$$\frac{\sigma_{x,max}}{\tau_{xy,max}} = 40$$

, thus, the maximum shear stress is much smaller than the maximum bending stress

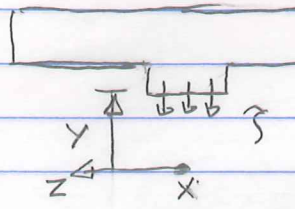
Example: I - section



Use for finding τ_{av} in right side of upper flange

Use for finding τ_{av} in web

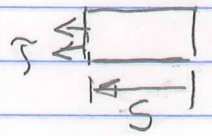
Use for finding τ_{av} in right side of lower flange.



$$Q = b t_f \left(\frac{h}{2} - \frac{t_f}{2}\right) + \left(\frac{h}{2} - t_f - y\right) t_w \cdot \frac{1}{2}\left(\frac{h}{2} - t_f + y\right)$$

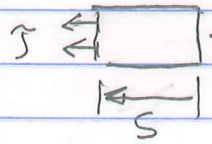
for $-\frac{h}{2} + t_f \le y \le \frac{h}{2} - t_f$

$\tau_{av} = VQ/I t_w \approx -\tau_{xy}$ in web



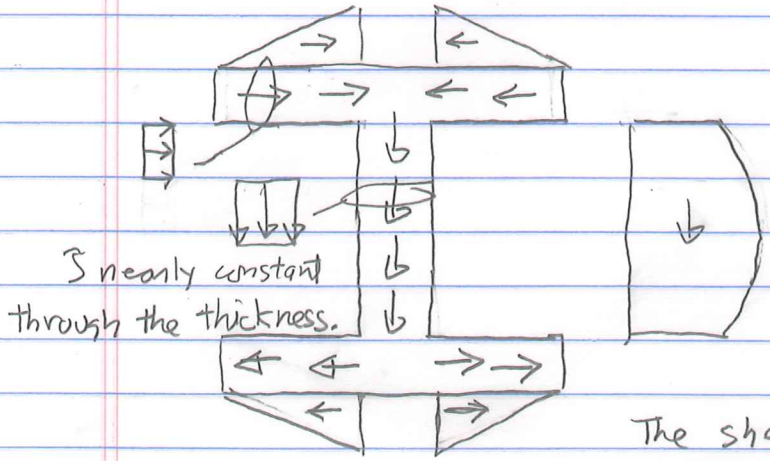
$Q = s t_f \left(\frac{h}{2} - \frac{t_f}{2}\right)$ for $0 \le s \le \frac{b}{2} - \frac{t_w}{2}$

$\tau_{av} = VQ/I t_f \approx \tau_{xz}$ in right side of upper flange



$Q = -s t_f \left(\frac{h}{2} - \frac{t_f}{2}\right)$ for $0 \le s \le \frac{b}{2} - \frac{t_w}{2}$

$\tau_{av} = VQ/I t_f \approx \tau_{xz}$ in right side of lower flange



Shows the direction and distribution of τ_{av} along the flanges and web. Through the thickness, the shear stress is nearly constant (but not near the flange-web junctions).

The shear stress in the web at $y = 0$ will be the maximum.